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HEAT TRANSFER BY COMBINED RADIATION AND CONDUCTION IN CRYOGENIC VACUUM-MULTILAYER THERMAL INSULATION

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We have obtained an approximate analytical solution of the problem of heat transfer by combined radiation and conduction in vacuum-multilayer thermal insulation. Results of the calculations are compared with calorimetric measurements.

Vacuum-multilayer thermal insulation (VMI) is one of the most effective materials for protecting cryogenic equipment against the influx of heat from the surroundings. It consists of a large number of shields which reflect infrared radiation and have a low emittance, placed in an evacuated insulating cavity of the article, and separated from one another by thermal-insulation spacers.

The difficulty in analyzing heat transfer in VMI results largely from the strong interaction of radiation and conduction. This interaction, whose existence was confirmed experimentally in [1], is due to the absorption of radiation by the spacing material.

Most papers devoted to this problem treated combined heat transfer in media such as gases, liquids, glass, etc., which absorb but do not scatter radiation [2, 3]. Combined heat transfer in media which both absorb and scatter radiation has received relatively little attention. Mainly numerical solutions of special problems were obtained by computer, and were not compared with experiments [4-6].

In the present paper we have obtained an approximate analytical solution of the problem of heat transfer by combined radiation and conduction in a slab of material which absorbs, emits, and scatters radiation isotropically, and is bounded by two opaque surfaces at temperatures T_1 and T_2. It is assumed that the properties of the medium do not depend on the wavelength of the radiation.

In this case heat transfer by combined radiation and conduction is described by a system of two linear integrodifferential equations:

$$\mu \frac{dI(\mu,\ \tau)}{d\tau} = -I(\mu,\ \tau) + \frac{\omega}{2}\int_{-1}^{+1} I(\mu',\ \tau)\, d\mu' + (1-\omega)\frac{\sigma_0 T^4}{\pi}, \tag{1}$$

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TABLE 1. Apparent Radiative Heat Flux in Scattering, Absorbing, and Transparent Media. T_1 = 170°K, T_2 = 100°K

Type of medium	q_r, W/m²			
	(15)*	(16)	(17)	(18)
Scattering ($\omega \to 1$, $\tau_0 = 1$)				
ε=0,015	0,39	0,34		
ε=1	26,0	24,2		
Absorbing ($\omega = 0$, $\tau_0 = 1$)				
ε=0,015	11,3		11,3	
ε=1	30,0		28,8	
Transparent ($\tau_0 \to 0$)				
ε=0,015	0,50			0,34
ε=1	32,0			42,4

*Equation No.

$$\frac{d}{dx}\left[-\lambda\frac{dT}{dx}+2\pi\int_{-1}^{+1}I(\mu,\ x)\,\mu d\mu\right]=0. \tag{2}$$

Using a modified method of discrete ordinates [6], which consists in replacing the integral terms by the Markov three-point quadrature formula, we obtain a system of three nonlinear equations which can be written in the following dimensionless form:

$$\frac{2}{3}\frac{dI_1}{d\tau}=-I_1+\frac{3\omega}{2(4-\omega)}(I_1+I_2)+\frac{1-\omega}{4-\omega}\frac{4\mathrm{Ki}}{\pi}\theta^4, \tag{3}$$

$$-\frac{2}{3}\frac{dI_2}{d\tau}=-I_2+\frac{3\omega}{2(4-\omega)}(I_1+I_2)+\frac{1-\omega}{4-\omega}\frac{4\mathrm{Ki}}{\pi}\theta^4, \tag{4}$$

$$\frac{d^2\theta}{d\tau^2}=-\frac{6\pi(1-\omega)}{4-\omega}(I_1+I_2)+\frac{3(1-\omega)}{4-\omega}4\mathrm{Ki}\,\theta^4, \tag{5}$$

where $\mathrm{Ki}=\frac{\sigma_0 T_1^3}{\lambda\gamma}$; $\theta=T/T_1$; $\tau=\gamma x$.

The dimensionless heat flux in the layer is

$$\Psi=\frac{q}{\lambda\gamma T_1}=\pi(I_1-I_2)-\frac{d\theta}{d\tau}\ . \tag{6}$$

Let us consider an element of VMI consisting of two shields and the spacer between them. In this case we can assume in the first approximation that the emissivities of the bounding surfaces are identical, i.e., $\varepsilon_1 \simeq \varepsilon_2 = \varepsilon$. Then the boundary conditions can be written in the form

$$\text{for}\quad \tau=0,\ \ \theta=1,\ I_1(0)=(1-r)\frac{\mathrm{Ki}}{\pi}+rI_2(0), \tag{7}$$

$$\text{for}\quad \tau=\tau_0,\ \theta=\theta_2,\ I_2(\tau_0)=(1-r)\frac{\mathrm{Ki}}{\pi}\theta_2^4+rI_1(\tau_0), \tag{8}$$

where $r = 1 - \varepsilon$.

To solve Eqs. (3)-(5) we linearize them by the substitution

$$\theta^4 \simeq 1+(\theta_2^4-1)\frac{\tau}{\tau_0} \tag{9}$$

Expression (9) corresponds to a linear distribution of the fourth power of the temperature over the thickness of the layer. It is characteristic for pure radiation transfer, and was taken because radiation plays a significant role in the total heat transfer in VMI.

TABLE 2. Results of a Study of Two-Shield Systems

Shields				Spacers						Heat flux, W			
ε	temp., °K			Material	No. pieces	Optical properties			Density of insulation ρ, shields/cm	calorimeter		$q_{r,opt}$ (15)	$k = \frac{q_{r,cal}}{q_{r,opt}}$
	T_1	T_2	T_r			ω	τ_0	γ, 1/m		q	$q_{r,cal}$		
1,0	125	98	112	Glass paper SBR-M	4	0,668	4,09	2045	5	2,80	2,57	2,30	1,12
1,0	181	137	160	"	4	0,624	3,91	1955	5	12,80	12,40	11,30	1,10
1,0	167	135	152	"	1	0,630	0,98	1970	20	16,50	15,40	15,70	0,98
1,0	103	90	99	"	1	0,678	1,03	2066	20	2,24	1,80	1,61	1,12
0,015	133	100	117	"	4	0,660	4,07	2035	5	2,53	2,25	2,07	1,10
0,015	172	115	145	"	4	0,640	3,96	1980	5	7,09	6,61	6,92	0,96
0,015	170	102	135	"	1	0,645	1,00	2000	20	6,30	4,00	4,47	0,90
0,040	225	100	170	Cellulose paper ODP	4	0,600	6,00	3540	6	24,40	21,30	23,50	0,91

The substitution of (9) into Eqs. (3)-(5) leads to a system of inhomogeneous linear equations. The general solution of system (3), (4) has the form

$$I_1 = C_1 A + C_2 A^{-1} + \frac{\mathrm{Ki}}{\pi}\left[1 + \frac{\theta_2^4 - 1}{\tau_0}\left(\tau - \frac{2}{3}\right)\right], \tag{10}$$

$$I_2 = B_1 C_1 A + B_2 C_2 A^{-1} + \frac{\mathrm{Ki}}{\pi}\left[1 + \frac{\theta_2^4 - 1}{\tau_0}\left(\tau + \frac{2}{3}\right)\right], \tag{11}$$

where $A = \exp\left(3\tau\sqrt{\frac{1-\omega}{4-\omega}}\right)$; $B_{1,2} = \frac{8 - 5\omega \pm 4\sqrt{(1-\omega)(4-\omega)}}{3\omega}$.

Substituting (10) and (11) into (5), using (9), and integrating twice, we have

$$\frac{d\theta}{d\tau} = -2\pi\sqrt{\frac{1-\omega}{4-\omega}}\,[C_1(1+B_1)A - C_2(1+B_2)A^{-1}] + C_3, \tag{12}$$

$$\theta = C_3\tau - \frac{2}{3}\pi\,[C_1(1+B_1)A + C_2(1+B_2)A^{-1}] + C_4. \tag{13}$$

After finding the integration constants by using boundary conditions (7) and (8), we obtain an expression for the dimensionless heat flux

$$\Psi = \frac{1-\theta_2}{\tau_0} + \frac{4}{3}\mathrm{Ki}\,\frac{1-\theta_2^4}{\tau_0}E, \tag{14}$$

where

$$E = 1 - \frac{1 + r\,\{A_0^{-1}[(r-B_2)A_0^{-1} - (1-rB_2)](1+B_1)(1-A_0) + \ldots}{3\tau_0\qquad\{(1-rB_1)(r-B_2)A_0^{-2} - \ldots}$$

$$\ldots \frac{+\,[(1-rB_1)A_0^{-1} - (r-B_1)](1+B_2)(1-A_0^{-1})\}}{-\,(1-rB_2)(r-B_1)\}},\quad A_0 = A \text{ for } \tau = \tau_0.$$

Substituting the expressions for Ψ, Ki, and θ_2 into (14), we obtain the heat flux density in the form

$$q = \frac{\lambda(T_1 - T_2)}{l} + \frac{4}{3}\,\frac{\sigma_0(T_1^4 - T_2^4)}{\tau_0}E, \tag{15}$$

where $l = \tau_0/\gamma$.

For a pure absorber ($\omega = 0$) the solution of the original system of equations leads to an analogous expression in which the function E is replaced by

TABLE 3. Apparent Radiation Conductivity of VMI for ε = 0.03 in the Temperature Range 300–90°K

Spacing material	ρ, shields/cm	λ_r, μW/cm°K calorimeter	(19)	(20)
Glass paper SBR-M	25	0,28	0,94	0,30
Glass-fiber mat ÉVTI-15	15	0,60	1,78	0,57
Cellulose paper ODP	22	0,45	1,38	0,44

$$F = 1 - \frac{1+r}{3\tau_0} \frac{(1-D_0^{-1})[2-(1+r)D_0^{-1}]}{1-rD_0^{-2}},$$

where $D_0 = \exp\left(\frac{3}{2}\tau_0\right)$.

As a result of the interaction of the radiation and conduction terms, the right-hand side of Eq. (15) should not be considered as conductive and radiative heat fluxes, but as components of the total heat flux proportional respectively to the difference of the first and fourth powers of the temperatures of the bounding surfaces. These terms can conventionally be called the apparent conductive and radiative heat fluxes.

The validity of Eq. (15) was confirmed by comparing it with known solutions for a pure scatterer, a pure absorber, and a transparent medium. The radiative component of the heat flux through a pure scatterer (ω = 1) is [7]

$$q_r = \frac{\sigma_0 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_{re}} + \frac{3}{4}\tau_0}. \tag{16}$$

Several solutions are known for a pure absorber. One of the most accurate [2] has the form

$$q_r = \frac{4}{3} \frac{\sigma_0 (T_1^4 - T_2^4)}{\tau_0} Y. \tag{17}$$

The values of the function $Y(\varepsilon, \tau_0)$ are tabulated in [2]. In the limiting case of a transparent medium ($\tau_0 = 0$) [7]

$$q_r = \varepsilon_{re}\sigma_0 (T_1^4 - T_2^4). \tag{18}$$

Table 1 lists calculated values of the apparent radiative heat flux from Eq. (15) in the immediate neighborhoods of the limiting values of the parameters ω and τ_0 for various emissivities of the bounding surfaces. These results are in satisfactory agreement with values calculated with Eqs. (16)-(18) for corresponding limiting cases. The greatest disparity occurs for values of $\tau_0 \ll 1$, which are practically never encountered in vacuum-multilayer insulation.

The validity of the solution obtained is confirmed also by a comparison of the values calculated with Eqs. (9) and (13) for τ_0 = 0-1, which showed that in spite of the substantial differences in the structure of these equations, values calculated with them agree to within 2 or 3% on the average.

Table 1 shows that radiation heat transfer is weakly dependent on the scattering albedo for ε = 1. The effect of ω increases appreciably with a decrease in emissivity of the bounding surfaces. This result agrees with numerical solutions obtained in [4]. Values calculated with Eq. (15) were also compared with calorimetric measurements of heat fluxes for two-shield systems.

In performing the measurements, the sample was placed between cold and hot walls of a slab calorimeter and loaded according to the average load from its own weight in an actual packet of insulation. The cold wall of the calorimeter was formed by the bottoms of the main and guard chambers, filled with a cryogenic liquid. The heat flux through the sample was determined by the amount of cryogenic liquid evaporated from the main chamber of the calorimeter. Heat flow along the layers of insulation through the ends of the sample was eliminated by using a guard ring with controllable heating. The stray influx of heat to the main calorimeter chamber was less than 0.01 W. The temperatures of the shields were measured with manganin-constantan thermocouples 0.1 mm in diameter with a 2 mm working length of the junction. The heat fluxes through two-shield systems were measured for various emissivities of the shields and various optical thicknesses of the spacers between them. Experimental data on the temperature dependence of the effective thermal conductivity of VMI [1] were used to separate the apparent radiation component from the total heat flux measured by the calorimetric method. This relation is approximated by a straight line in the coordinates λ_{eff} and $(T_1+T_2)(T_1^2+T_2^2)$. The point of intersection of this straight line with the axis of coordinates for a zero value of the temperature combination gives the value of the apparent thermal conductivity of the solid, and its slope gives the apparent radiation conductivity. The optical characteristics of the spacing materials were determined by infrared spectroscopy [8].

The characteristics and results of the study of two-shield systems are listed in Table 2. A comparison of the data obtained shows that the values of the apparent radiative heat fluxes calculated with Eq. (15) agree with the calorimetric measurements to within 12%. Thus, Eq. (15) adequately describes heat transfer in a system consisting of two shields separated by a spacing material which absorbs and scatters infrared radiation.

We turn now to an analysis of heat transfer in multishield systems, i.e., in VMI, by combined radiation and conduction. For a system consisting of shields with spacers between them, and bounded by black walls, we can write

$$q_{i-(i+1)} = \frac{\lambda}{l}(T_i - T_{i+1}) + \frac{4}{3}\frac{\sigma_0(T_i^4 - T_{i+1}^4)}{\tau_0}E,$$

$$q_{(i+1)-(i+2)} = \frac{\lambda}{l}(T_{i+1} - T_{i+2}) + \frac{4}{3}\frac{\sigma_0(T_{i+1}^4 - T_{i+2}^4)}{\tau_0}E,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$q_{(N-1)-N} = \frac{\lambda}{l}(T_{N-1} - T_N) + \frac{4}{3}\frac{\sigma_0(T_{N-1}^4 - T_N^4)}{\tau_0}E.$$

Summing these expressions, and using the fact that

$$q_{i-(i+1)} = q_{(i+1)-(i+2)} = \cdots = q_{(N-1)-N} = q,$$

and that the two parts between the bounding walls and the adjoining shields can be considered as one part between two shields, we obtain

$$q = \frac{\lambda}{lN}(T_1 - T_2) + \frac{4}{3}\frac{\sigma_0(T_1^4 - T_2^4)}{N\tau_0}E.$$

Multiplying both sides of this equation by $\delta/(T_1 - T_2)$, and using the fact that $N/\delta = 1/l = \rho$, we obtain an expression for the effective thermal conductivity of VMI in the form

$$\lambda_{eff} = \lambda + \frac{4}{3}\frac{\sigma_0(T_1+T_2)(T_1^2+T_2^2)}{\rho\tau_0}E, \tag{19}$$

where the second term on the right-hand side is the apparent radiation conductivity of the insulation.

Table 3 lists the calculated values of these conductivities for VMI of several different structures with different spacing materials of glass and cellulose fibers compared with experimental calorimetric data [1] obtained in the temperature range 300-90°K. The table shows that in spite of substantial differences in structure and optical properties of the spacing materials, the VMI structures investigated are characterized by nearly the same values of the ratio $k = (\lambda_{r,cal}/\lambda_{r,opt}) < 1$ (the maximum deviation from the average values does not exceed 7%), which can be considered an empirical constant. Table 2 shows that for two-shield systems this constant is close to unity. This is due to the fact that Eq. (19), derived by a formal transformation from a two- to a multishield system, cannot reflect all the complexities of radiative heat-transfer processes in VMI. In particular, when radiation propagates in a layer of insulation, there is a considerable change in its spectral composition as a result of the variation of the temperatures of the shields from layer to layer, which also means a change in the optical properties of the spacing materials. In addition, the change in temperatures of the shields leads to a change in their emissivities.

By introducing the factor k into Eq. (19), we obtain the semiempirical relation

$$\lambda_{\text{eff}} = \lambda + k\frac{4}{3}\frac{\sigma_0(T_1+T_2)(T_1^2+T_2^2)}{\rho\tau_0}E, \tag{20}$$

where the empirical factor k = 0.32 for a packet of insulation with a large number of shields. Table 3 shows that the values calculated with Eq. (20) agree with the calorimetric data to within 5-7%. If the spacing material is a pure absorber, the function E in Eq. (20) must be replaced by the function F, as indicated above. Calculations show that neglecting scattering leads to an increase of the deviation of $\lambda_{r,cal}/\lambda_{r,opt}$ from the average value of up to 100% as a result of the difference of the scattering contribution to the attenuation of radiation by different materials.

Thus, the semiempirical relation (20) adequately describes heat transfer in VMI by combined radiation and conduction, and permits not only a qualitative, but also a quantitative estimate of the effect on it of the optical properties of spacing materials, and gives a basis for selecting spacing materials for the development of new efficient VMI composites.

NOTATION

I, integrated radiation intensity; μ, cosine of angle between ray and normal to boundary surface; τ_0, optical thickness; ω, scattering albedo; γ, integrated hemispherical total attenuation coefficient; σ_0, Stefan–Boltzmann constant; T, running temperature; T_1, T_2, boundary temperatures; θ, dimensionless temperature; Ki, Kirpichev number; λ, thermal conductivity; l, distance between shields; δ, thickness of insulation; N, number of shields; ρ, density of insulation; q, heat flux; x, running coordinate. Indices: eff, effective; cal, calorimetric; opt, optical; r, radiation; re, reduced.

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